SIMULTANEOUS-MOVE GAMES I

are:

- **Discrete, “pure” strategies (no dice-throwing)**
- **Either at the same time, or**
  without knowledge of an action already taken.

∴ **Imperfect information or knowledge**

e.g. **Choice of product design, advertising campaign, features**

e.g. **Goalie v. striker; server v. receiver**

Contents of This Lecture

1. The Payoff Matrix
2. Nash Equilibrium (N.E.)
3. The Prisoner's Dilemma
4. Four Methods for Finding the N.E.
    - Each has a dominant strategy
    - One has a dominant strategy
    - Eliminate dominated strategies
    - Best-response analysis
5. Other Games
6. Four Lessons

How To Avoid Circularity?

There is circularity:

I'm deciding what to do, while you are too;

what I decide affects you, and

what you decide affects me.

How to decide what to do?

1. **Write down the payoff matrix (or game table) which shows all outcomes for each of us for all combinations of actions, and**
2. **Look for a no-regrets combination of actions, yours and mine —**

   — A Nash Equilibrium.

Payoff Matrix (or "normal" or "strategic" form)

- Dimensions = number of players, here = 2.
- # rows = # strategies of Mr Row = 4.
  # columns = # strategies of Ms Column = 3.

Column

Row		Le	Ce	Ri
	T	3, 1	2, 3	10, 2
	H	4, 5	3, 0	6, 4
	L	2, 2	5, 4	12, 3
	B	5, 6	4, 5	9, 7

- *Non-zero sum* (or "positive sum") game:
  the sum of the payoffs is not constant across cells.
  By convention, the payoffs are: R,C.
- (See solution on page 6 below.)

A Zero-Sum Payoff Matrix

Gridiron football:

		Defense		
		Run	**Pass**	**Blitz**
Offense	**Run**	2	5	13
	Short pass	6	5.6	10.5
	Medium pass	6	4.5	1
	Long pass	10	3	−2

- **Show the payoffs of one player only (here, Offense).**
- **Payoffs in yards gained by Offense. (Defense loses that amount.)**
- **N.E. at {Short pass, Pass}, and Offense gains 5.6 yards.**

Nash Equilibrium

From p.4, N.E. at {L,M}, payoffs (5,4):

Column

Row

	Le	Ce	Ri
T	3, 1	2, 3	10, 2
H	4, 5	3, 0	6, 4
L	2, 2	5, 4	12, 3
B	5, 6	4, 5	9, 7

Why? Because Ce is Column's *best response* to Row's L, and vice versa.

So (L,Ce) is each player's best response to the other's action.

∴ Neither would change unilaterally.

∴ we have an equilibrium (a N.E.).

Further on N.E. (Nash Equilibria)

1. Look at strategies (H,Le), payoffs (4,5): Is this an equilibrium? No: if Column chooses Le, then R chooses B, because 5 > 4. But (B,Le) is not an equilibrium: Column chooses Ri (7 > 6). Etc.
2. The N.E. need not be the jointly best combination of strategies: strategies (B,Ri) has payoffs of (9,7), but it's not an equilibrium, absent "cooperative" behaviour.
3. Nor does the N.E. require equilibrium choices to be *strictly* better than other choices: if (B,Ce) had payoffs of (5,5), then (L,Ce) would remain a N.E. because Row has no incentive to change her choice from L to B.
4. Could do *cell-by-cell inspection* to find all N.E., but simpler methods exist.

N.E. as Beliefs

Players need not have best responses to opponents' action which have not yet happened.

Players can think ahead, and form *beliefs* of what opponents will do.

Then a N.E. can be defined as a set of strategies (one per player) such that:

1. **each player has correct beliefs about the strategies of the others, and**
2. **the strategy of each is the best strategy for herself, given her beliefs about the others' strategies.**

Now: Four Methods to Find N.E.

Say you're the Row player:

1. **Look for a *dominant strategy* (a row always preferred, no matter which column the other player chooses), and choose it.**
2. **Does the other player have a *dominant strategy* (column)? If so, expect that strategy.**
3. **Look for *dominated actions* (rows never preferred, no matter what the other player would choose), and *eliminate* them.**

   Successively eliminate each other's dominated strategies (rows, columns).
4. **Use arrows for both of you, and identify any cells with no arrows leaving: *best response* or N.E.**

1. Dominance

Consider the Prisoner's Dilemma:

Years of prison (Ned, Kelly).

		Kelly	
		Spill (D)	**Mum (C)**
Ned	**Spill (D)**	**8, 8**	**0, 20**
	Mum (C)	**20, 0**	**1, 1**

- **Spill the beans (Defect) is better than keeping Mum (Cooperate) for Ned, whatever he believes Kelly will do.**

∴ **Spill is a *dominant strategy* for Ned, and Mum is a *dominated* strategy.**

- **Likewise for Kelly.**

Both Players Have Dominant Strategies

- **In the Prisoner's Dilemma, both players have a *dominant strategy*: no matter what the other guy does, Spill the beans (or Defect) is best.**

∴ **N.E. at {Spill, Spill}, with payoffs of (8, 8) years of prison.**

- **Each player would have preferred (Mum, Mum), with payoffs of (1, 1) years, but without cooperation (or trust, or ?) it's unattainable.**
- **(See Lectures 15, 16 later.)**

Prisoner's Dilemmas have three necessary characteristics:

1. Two strategies: Cooperate C with the other player, or Defect D (here, Mum and Spill);
2. Each player has a dominant strategy: Defect; and
3. The N.E. of (D,D) is worse for both players than (C,C):
   here (D,D) $\rightarrow$ (8,8); while (C,C) $\rightarrow$ (1,1)

   That is, there is a conflict between collective interest (at C,C) and individual self-interest (at D,D).

Many real-world phenomena are PDs. Examples?

How to overcome the (D,D) trap?

(See Lectures 15, 16 later.)

Ex: The Advertising Game is a P.D.

Case: Telstra and Optus and advertising.

David Ogilvy: ***Half the money spent on advertising is wasted; the problem is identifying which half.***

Telstra and Optus independently must decide how heavily to advertise.

Advertising is expensive, but if one telco chooses to advertise moderately while the other advertises heavily, then the first loses out while the second does well.

Payoffs in the Telstra/Optus Game:

Let's assume if both Advertise Heavily then Telstra nets $70,000, while Optus nets $50,000.

But if Telstra Advertises Heavily while Optus Advertises Moderately only, then Telstra nets $140,000 while Optus nets only $25,000, and vice versa.

If both Advertise Moderately, then Telstra nets $120,000 and Optus nets $90,000.

What to do?

Consider the payoff matrix:

The Advertising Game

		Optus	
		Heavy	**Moderate**
Telstra	**Heavy**	70, 50	140, 25
	Moderate	25, 140	120, 90

Both choose Heavy advertising, although each would be better off with Moderate advertising.

A Prisoner's Dilemma.

The arrows show each player has a dominant strategy of H.

With Pure Strategies, Rankings are Sufficient:

Or, could rank outcomes for each player:
4 is best, 1 is worst.

		Optus	
		Heavy	**Moderate**
Telstra	**Heavy**	**2, 2**	**4, 1**
	Moderate	**1, 4**	**3, 3**

Important: When strategies are "pure" (deterministic), then we needn't have exact knowledge of the payoffs, just their rankings.

Ex: The Capacity Game

Players: two firms Alpha and Beta

Strategies:
Allow three choices for each of the two players:

- Do Not Expand production capacity (DNE),
- Small expansion, and
- Large expansions.

$\therefore$ A 3×3 payoff matrix (POM)

Greater capacity $\rightarrow$ more sales $\rightarrow$ lower prices. Profits?

The payoff matrix (in net returns '000) for simultaneous moves is:

The Capacity Game

		Beta DNE	*Beta* Small	*Beta* Large
Alpha	DNE	$18, $18	$15, $20	$9, $18
Alpha	Small	$20, $15	$16, $16	$8, $12
Alpha	Large	$18, $9	$12, $8	$0, $0

The payoff matrix (Alpha, Beta).

N.E. at {Small, Small}, although both would prefer {DNE, DNE}.

Large is a dominated strategy for both players.

What if the payoffs were the *differences* in returns? Then the game is changed.

2. One Player Has a Dominant Strategy

		RBA	
		Low	High
Gov't	Balanced	3, 4	1, 3
	Deficit	4, 1	2, 2

Players:
Gov't: fiscal policy (taxes, govt. expenditure)
RBA: monetary policy (interest rates)

Actions:
Gov't: either balanced budget or deficit
RBA: high or low interest rates

Preferences?

Ex: The Macroeconomics Game

The RBA's best strategy depends on the Gov't's strategy. Dislikes inflation, High rates.
The Gov't prefers spending (and a budget deficit).

The RBA realises that {Deficit} is a dominant strategy for Gov't.
$\therefore$ RBA should choose {High}.

$\therefore$ Payoffs of (2,2), although {Balanced, Low} $\rightarrow$ (3,4) is jointly better.

Many countries have a loose fiscal policy and a tight monetary policy at {Deficit, High interest rates}.

3. Successive Elimination of Dominated Strategies

Column

Row		Le	Ce	Ri
	T	3, 1	2, 3	10, 2
	H	4, 5	3, 0	6, 4
	L	2, 2	5, 4	12, 3
	B	5, 6	4, 5	9, 7

For Row, H is dominated (by B): eliminate H;
For Column, Le is dominated (by Ri);
For Row, T and B are now dominated (by L).
Which now leaves Row with L, and Column chooses Ce.

Not every game is *dominance solvable*, but the POM perhaps becomes smaller.

What if there are ties?

It's possible to eliminate using *weak dominance* ($\leq$) instead of *strict dominance* ($<$), but this successive elimination of weakly dominated strategies might throw out some N.E.

(See Dixit & Skeath, p. 97.)

4. Best-Response Analysis (BRA) to Find N.E.

Dixit & Skeath use circles to show the best response. Row looks at for highest payoff in each column, and Column looks for the best payoff in each row.

Here I use arrows (in columns and rows, respectively): pointing to the best response, and showing which strategies would not be chosen.

Then: N.E. are cells with arrows only entering, not leaving: need entering arrows from each dimension (2 for a 2-person interaction).

When BRA fails to find a N.E., there is no N.E. in pure strategies (but see Lecture 11).

In Lecture 5, we derive best-reponse curves with continuous strategies.

Pure Coordination Games

Common interests, but independent choices → issues.

Harry \ *Sally*	Starbucks	Local
Starbucks	1, 1	0, 0
Local	0, 0	1, 1

Two N.E., with equal payoffs: need to coordinate.

How? Without communication, to a *focal point.*

Assurance Games

		Sally Starbucks	Local
Harry	Starbucks	1, 1	0, 0
	Local	0, 0	2, 2

Now a shared preference for the Local, over Starbucks.

This needs to be *common knowledge*.

But also need a *convergence of expectations* of actions.

Need enough certainty or *assurance* to get to (Local, Local).

The Battle of the Sexes

A *coordination game*:

e.g. video VHS v. Sony's Betamax;

now the competing standards for digital audio disks:
SACD (Sony & Philips) v. DVD-A (Toshiba, Matsushita, Pioneer etc.)

and DVD recording: DVD+R, DVD-R, DVD-RAM.

and the high-definition DVD:
Blu-ray DVD v. HD-DVD.

The Players & Actions:

- **a man (Hal) who wants to go to the Theatre and**
- **a woman (Shirl) who wants to go to a Concert.**

While selfish, they are deeply in love, and would, if necessary, sacrifice their preferences to be with each other.

The payoff matrix (measuring the scale of happiness) is below.

What are all equilibria?

(i.e. Which pairs of actions are mutually best response?)

The Battle of the Sexes

		Shirl	
		Theatre	**Concert**
Hal	**Theatre**	**2, 1**	**−1, −1**
	Concert	**−1, −1**	**1, 2**

The payoff matrix (Hal, Shirl).

A non-cooperative, positive-sum game, with two Nash equilibria.

The Battle of the Sexes

There is no iterated dominant strategy equilibrium.

There are two *Nash equilibria*:

- **(*Theatre*, *Theatre*): given that Hal chooses *Theatre*, so does Shirl.**
- **(*Concert*, *Concert*), by the same reasoning.**

How do the players know which to choose?

(A coordination game.)

Players' choices.

If they do not talk beforehand, Hal might go to the Concert and Shirl to the Theatre, each mistaken about the other's beliefs.

Focal points?

Repetition?

Each of the Nash equilibria is collectively rational (efficient): no other strategy combination increases the payoff of one player without reducing that of the other.

Market analogues ?

- **Battle over an industry-wide standard.**
- **The choice of language used in a contract when two firms want to formalise a sales agreement but prefer different terms.**
- **Bought a DVD player recently?**
  DVD, CDV, MP3, CD, DVD+, etc.
  Digital audio disks: SACD (Sony & Philips) v. DVD-A (Toshiba, Matsushita, Pioneer)
  Emerging standards mean choice and decisions for early adopters.
- **others?**

No Equilibrium in Pure Strategies?

		Serena DL	*Serena* CC
Venus	DL	50	80
Venus	CC	90	20

Zero-sum game: Venus's percentage of wins against Serena.

Play Down the Line, or Cross Court.

∴ No N.E. in pure strategies. Why?

(See Lecture 11 later.)

Chicken!

Here "Bomber" and "Alien" are matched.

		Bomber Veer	*Bomber* Straight
Alien	Veer	Blah, Blah	Chicken!, Winner
Alien	Straight	Winner, Chicken!	Death? Death?

No dominant strategies: what's best for one depends on the other's action.

Nash Equilibrium where?

Six Steps to Help:

1. **What is the strategic Issue?**
2. **Who are the Players?**
3. **What are each player's strategic Objectives?**
4. **What are each player's potential Actions?**
5. **What is the likely Structure of the game?**
   — simultaneous or sequential (who's on first?)?
   — one-shot or repeated?
6. **Simultaneous: Rank each player's Outcomes across all combinations of the actions of both.**

What Have We Learnt?

Rule 1: **Look ahead and reason back.**

Rule 2: **If you have a dominant strategy, then use it.**

Rule 3: **Eliminate any dominated strategies from consideration, and go on doing so successively.**

Rule 4: **Look for an equilibrium, a pair of strategies in which each player's action is the best response to the other's.**

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