SEQUENTIAL-MOVE GAMES

Strategic interactions in which there is:
a clear order of play, or an option of moving first.

Players take turns, and know what's happened.

Players *look forward and reason back:*
"If I do this, how will my opponent respond?"

Q: When is it to a player's advantage to move *first*, and when *second*? Or last?

Players can devise *strategic moves* to manipulate the order of play to their advantage; see Lecture 14.

Game Trees (or extensive forms)

Use a *game tree*, in which the players, their actions, the timing of their actions, their information about prior moves, and all possible payoffs are explicit.

Use *nodes* (or action or decision nodes) and *branches*.

Any order and number of consecutive moves per play allowed.

Mother Nature may reveal her hand, too (chance nodes, with uncertainties).

Can trace different paths from the *initial node* to final payoffs at a *terminating node*.

Boeing v. Airbus

Airbus and Boeing will develop a new commercial jet aircraft.

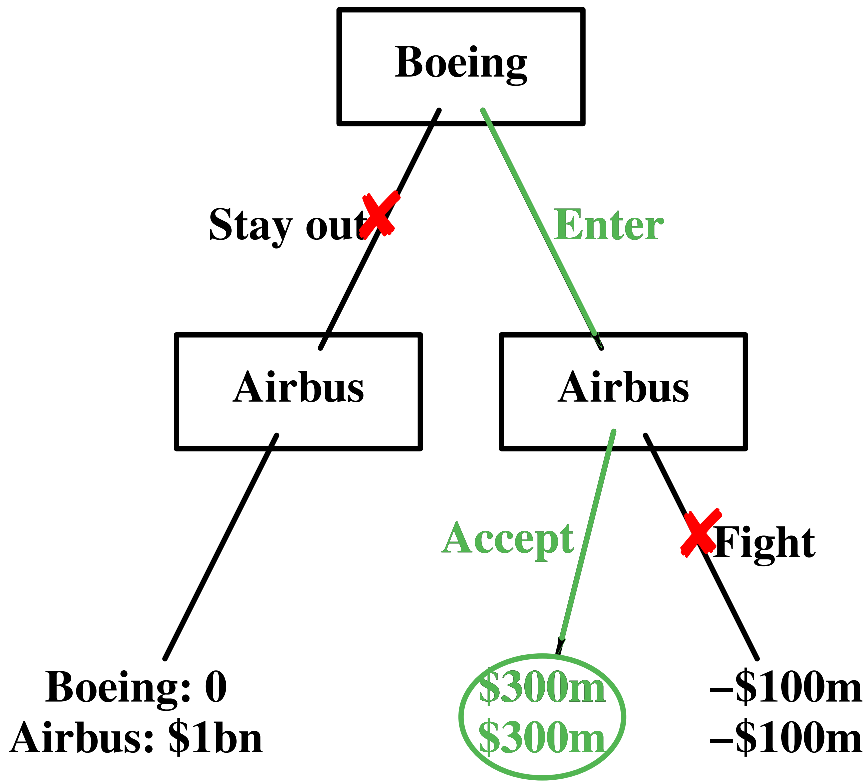
Airbus is ahead, and Boeing is considering whether to enter the market.

If Boeing stays out, it earns zero profit, while Airbus enjoys a monopoly and earns a profit of $1 billion.

If Boeing enters, then Airbus has to decide whether to accommodate Boeing peacefully, or to wage a price war.

With peace, each firm will make a profit of $300 m.
With a price war, each will lose $100 m.

A Game Tree



Q: How should Airbus respond?
∴ What should Boeing do?

Rollback, or Backwards Induction

1. From the terminal nodes (final payoffs), go up the tree to the first parent decision nodes.
2. Identify the best decision for the deciding player at each node.
3. "Prune" all branches from the decision node in 2. Put payoffs at new end = best decision's payoffs
4. Do higher decision nodes remain? If "no", then finish.
5. If "yes", then go to step 1.
6. For each player, the collection of best decisions at each decision node of that player → best strategies of that player.

The Capacity Game Revisited

In lecture 2 the two firms Alpha and Beta simultaneously made the capacity decision:

		Beta	
	DNE	Small	Large
DNE	\$18, \$18	\$15, \$20	\$9, \$18
Alpha Small	\$20, \$15	\$16, \$16	\$8, \$12
Large	\$18, \$9	\$12, \$8	\$0, \$0

N.E at (Small, Small).

Q: What if Alpha moved first?

The game tree, and first-mover advantage.

If Alpha preempts Beta, then use the *game tree*:

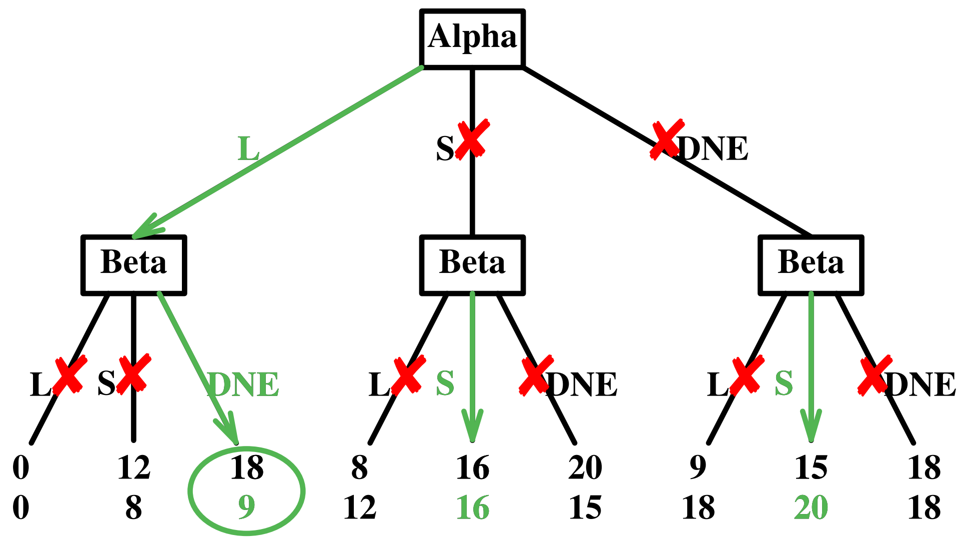


Figure 1. Game Tree, Payoffs: Alpha's, Beta's

N.E. at Alpha: Large; Beta: Don't Expand, ({L,DNE}, not {S,S}).
Payoffs now ($18, $9) instead of ($16, $16):
Alpha is $2 better off, but Beta is $7 worse off.

The game and its N.E. have changed. Why?

Because of Commitment...

- **In the simultaneous game Large is dominated for Alpha: Alpha will never use it. (The simultaneous game is really a P.D.) So the equilibrium outcome is Alpha: Small; Beta: Small.**
- **In the sequential game (see the game tree above) Alpha's strategic move is to preempt Beta by unconditionally choosing Large. So the equilibrium outcome is Alpha: Large; Beta: Do Not Expand.**
- **In the sequential game, Alpha's capacity choice has *commitment value*: it gives Alpha (in this case) *first-mover advantage*. Alpha can benefit from limiting its freedom and taking an irreversible action.**

Order Advantages

In the Capacity Game, Alpha gains $2 by preempting Beta: *first-mover advantage*.

In some interactions, however, there is *second-mover advantage*: a catalogue company whose catalogue came out later could undercut its rival whose catalogue and prices were announced first.

First-mover advantage comes from the ability to *commit* to an advantageous position and to force other players to adapt to it.

Second-mover advantage comes from the *flexibility* to adapt oneself to the others' choices.

Commitment v. flexibility?

Evidence of Rollback

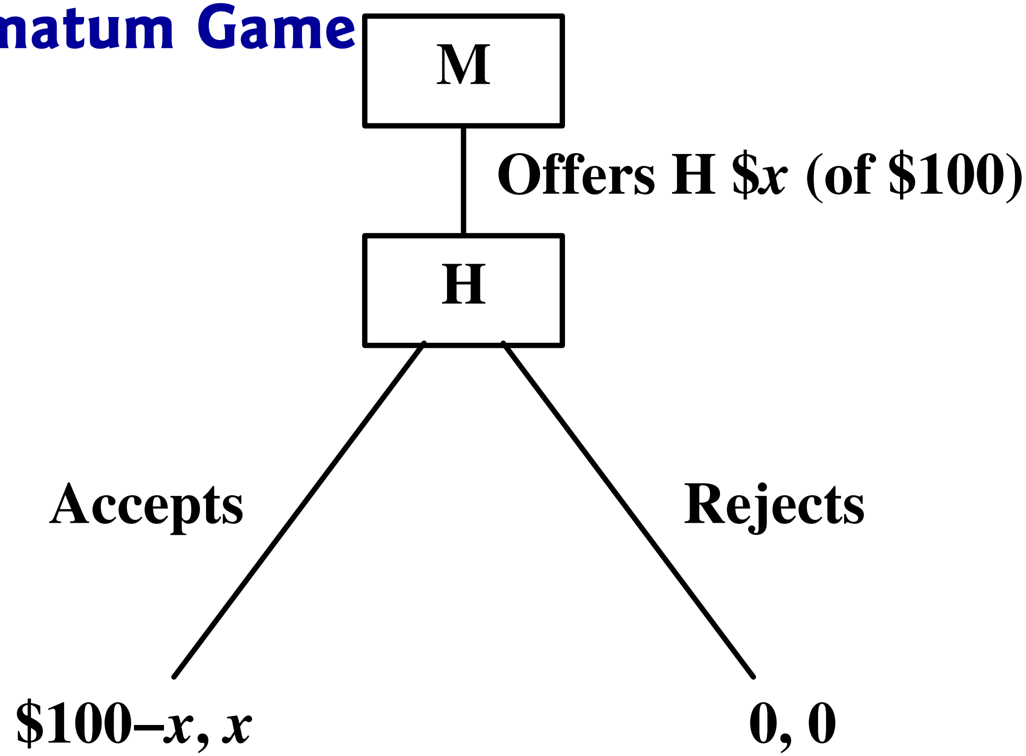
The Ultimatum Game:

***Players and game*: Mortimer and Hotspur are to divide \$100 between themselves. The game structure is common knowledge.**

- **Mortimer offers Hotspur an amount \$$x$ of the \$100. Then —**
- ***either* Hotspur accepts \$$x$, and Mortimer receives the remainder of the \$100, and the game ends;**
- ***or* Hotspur rejects \$$x$, and neither gets anything.**

What will Mortimer offer \$$x$?
What would you offer? (Write it down.)

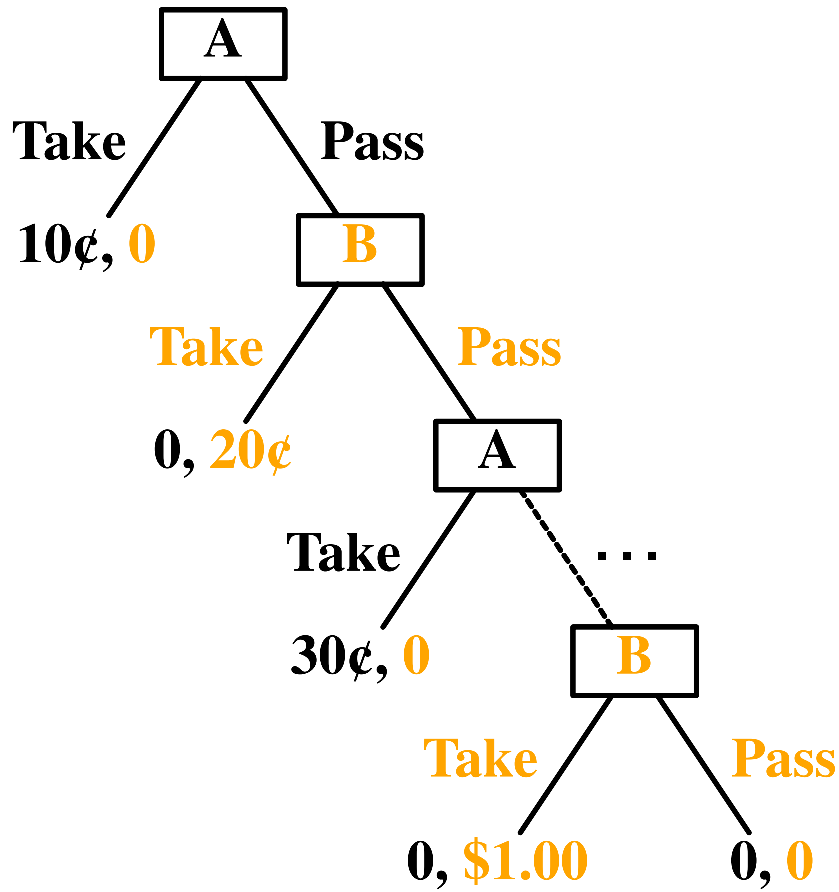
The Ultimatum Game



Most offer a 50:50 split, and almost all accept.
Most reject less than $25 offered, and some even $40.

A fairness (equal) focal point.

The Centipede Game



What would you do: as A? as B?

The Centipede Game

Since B will take the \$1 at the last stage, A should take 90¢ at the second-last stage. Since A would take the 90¢ at the second-last stage, B should take 80¢ at the third-last stage. Etc.

So A should take the 10¢ to begin with.

But often goes for a few rounds (apparently irrationally).

Why?

Perhaps players care not only about \$ and ¢, but also about fairness or reputation.

∴ Don't assume that the other player (whether an acquaintance or anonymous or new) has your values.

THREE CLASSROOM INTERACTIONS

I. Auctioning a Ten-Dollar Note

Rules:

- **First bid: 20¢**
- **Lowest step in bidding: 20¢ (or multiples of 20¢)**
- **The auction lasts until the clock starts ringing.**
- **The highest bidder pays bid to auctioneer and gets \$10 in return.**
- **The second-highest bidder also pays her bid to auctioneer, but gets nothing.**

The Ten-Dollar Auction

Write down the situation as seen by

1. **the high bidder, and**
2. **the second highest bidder.**

What happened?

Escalation and entrapment

Examples?

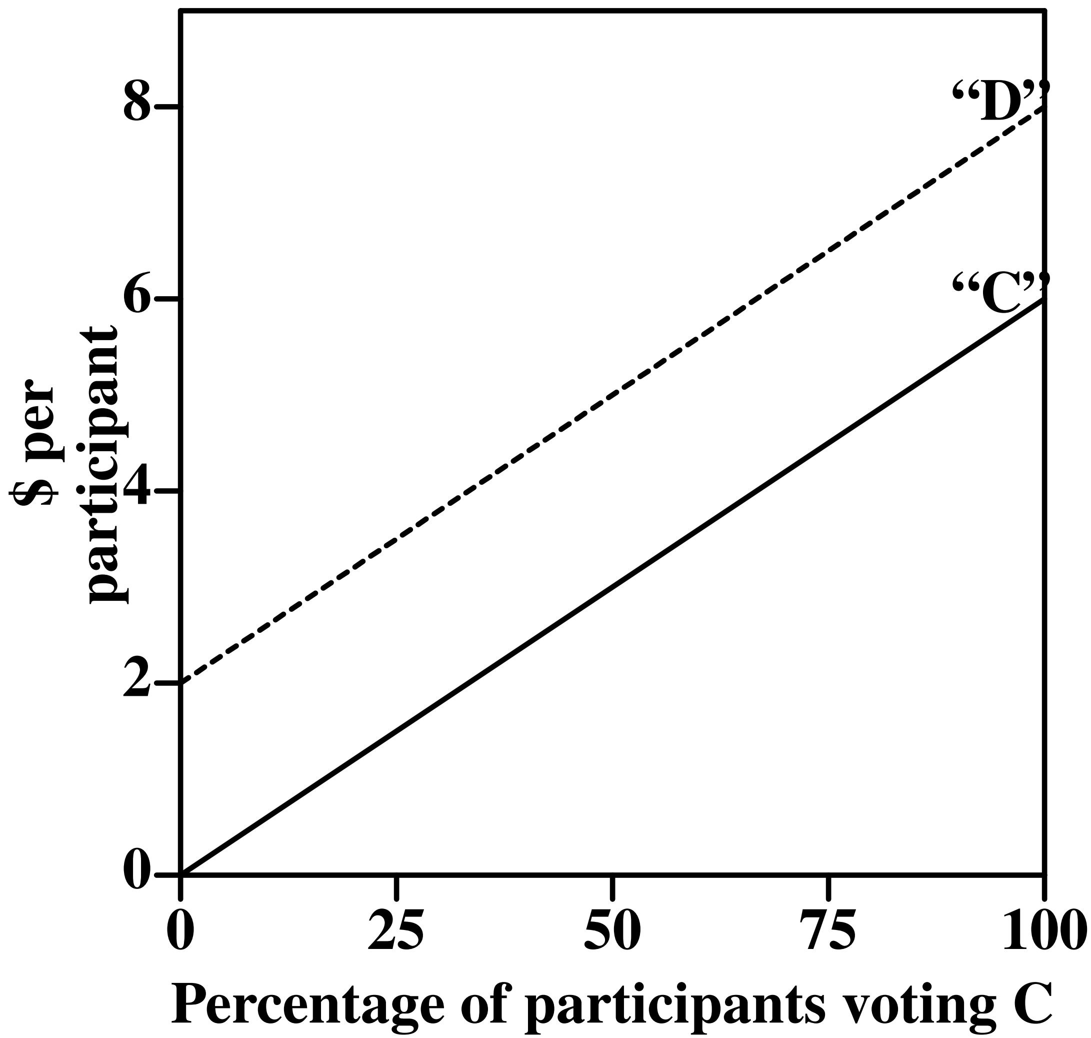
(See O'Neal's article in the Readings.)

II. Schelling's Game

Rules:

- ➢ Single play, \$4 to play: by writing your name on the slip
- ➢ Vote "C" (Coöperate) or "D" (Defect).
- ➢ Sign your ballot. (and commit to pay the entry fee.)
- ➢ If x% vote "C" and $(100 - x)$% vote "D":
  - then "C"s' net payoff = $\left(\frac{x}{100} \times \$6\right) - \$4$
  - then "D"s' net payoff = "C" payoff + \$2
- ➢ Or: *You* needn't play at all.

Schelling's Game



Note: the game costs $4 to join.

Schelling's Game

What happened?

- **numbers & payoffs.**
- **previous years?**

Dilemma: $\left\{\begin{array}{l}\textbf{coöperate for the common good } \textit{or} \\ \textbf{defect for oneself}\end{array}\right.$

Public/private information

Schelling's n-person Game

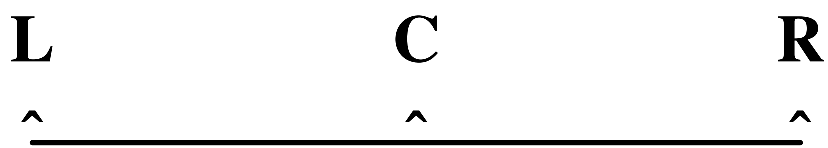
Examples?

- price
- tax avoidance
- individual negotiation
- coal exports
- market development
- others?

(See Schelling in the Package.)

III. The Ice-Cream Sellers

(See Marks in the Web page.)



- **Demonstration**
- **Payoff matrix**
- **Incentives for movement?**
- **Examples?**

Modelling the ice-cream sellers.

We can model this interaction with a simplification: each seller can either:

- move to the centre of the beach (M), or
- not move (stay put) (NM).

The share of ice-creams each sells (to the total population of 80 sunbathers) depends on its move and that of its rival.

Each sunbather buys one ice-cream, from the closer seller.

Since each has two choices for its location, there are $2 \times 2 = 4$ possibilities.

The Sellers’ Payoff Matrix

		The Other Seller	
		M	**NM**
You	**M**	**40, 40**	**50, 30**
	NM	**30, 50**	**40, 40**

The payoff matrix (You, Other).

A non-cooperative, zero-sum game, with a *dominant strategy*, or dominant move.

Real-World Ice-Cream Sellers

Think of the beach as a product spectrum, each end representing a particular niche, and the centre representating the most popular product.

Demand is largest for the most popular product, but so is competition.

This simple model: a tendency to avoid extremes, especially with barriers to entry for new players.

Examples:

- the convergence of fashions?
- the similarity of commercial TV and radio programming?
- the copy-cat policies of political parties?
- the parallel scheduling of Qantas/JetStar and Virgin Blue?

A twist: What if the centre is too far for some bathers (at the ends of the beach) to walk?

Then the tendency for the sellers to offer the same product (at the centre) is reduced, and they might differentiate their products.

Seven issues addressed in Game Theory:

1. What does it mean to choose strategies "rationally" when outcomes depend on the strategies chosen by others and when information is incomplete?
2. In "games" that allow mutual gain (or mutual loss) is it "rational" to cooperate to realise the mutual gain (or to avoid the mutual loss) or is it "rational" to act aggressively in seeking individual gain regardless of mutual gain or loss?
3. If the answers to 2. are "sometimes," then in what circumstances is aggression rational and in what circumstances is cooperation rational?

4. **In particular, do continuing relationships differ from one-off encounters (one-night stands?) in this issue?**
5. **Can moral rules of cooperation emerge spontaneously from the interactions of rational egoists?**
6. **How well does actual human behaviour correspond to "rational" behaviour in these cases?**
7. **If it differs, then how? Are people more cooperative than would be "rational?" More aggressive? Both?**

Cooperative and Non-cooperative Games

Question 1:

A wholesaler wants to merge with any one of four retailers who jointly occupy a city block. If the merger goes through, the wholesaler and the retailer will make a combined profit of \$10 million.

The retailers have an alternative: they can band together and sell to a real estate company, making a joint profit of \$10 million that way.

Can the outcome be predicted?

If the wholesaler joins a retailer, how should they divide the \$10 million?

Question 2:

An inventor and either of two competing manufacturers can make \$10 million using the inventor's patent and the manufacturer's factory.

If the inventor and one of the manufacturers should manage to get together, how should they share their profit?

— both examples of *Cooperative Games*

with

- **— agreements binding on all players, and**
- **— means of transferring payoffs between players.**

(See Dixit & Skeath, Chapter 17.)
But for SGTM: Non-Cooperative Game Theory only

Cooperative game theory:

what kinds of *coalitions* a group of players will form:

- **if different coalitions produce different outcomes?**
- **if these joint outcomes have to be shared among members?**

***Non-cooperative game theory*:**

no binding agreements,
and which *strategies* will players choose?

Where Are We?

1. Strategic interactions.
2. Look forward and reason backwards.
3. Simple games.
4. Payoff matrix (strategic or normal form), and arrows
5. Dominant strategy; iterated dominant strategy.
6. Nash equilibrium.
7. Pure v. mixed strategies. (more later)
8. Extensive-form game tree for sequential games; rollback, (information sets — later).