CONTINUOUS STRATEGIES: OLIGOPOLIES

Oligopolies are markets with a few sellers only.

Even if they are not selling perfect substitutes, the actions of each (price, quantity) can affect the profits of the others: a strategic interaction.

We can use three strategic game-theoretic models:

1. von Stackelberg leader/follower competition
2. Cournot simultaneous *quantity* competition
3. Bertrand simultaneous *price* competition

Plus:

4. Strategic Complements and Substitutes
5. Benchmarking market performance.

1. A Von Stackelberg Leader-Follower Model.

e.g., duopoly — two sellers — with a dominant firm (once IBM, now Microsoft, or OPEC, etc.)

Model: □ **Leader (Spring) produces quantity Q_S of bottled water, and**

□ **Follower (Crystal) produces Q_C of identical water: a homogeneous good.**

- **market demand → price P (i.e. equal for both)**

$$P = 10 - (Q_S + Q_C), \;\; Q_S + Q_C \leq 10.$$

- **common knowledge: Spring sets its production level before Crystal does**
- **each firm's production level: common knowledge**
- **order of play & output choices: common knowledge**
- **no capacity constraints**

Let's put numbers in:

- **each firm has identical costs ($i = C, S$):**

$$MC_i = AC_i = \$3/\text{unit}$$

∴ **payoffs for either firm i are ($i = C, S$):**

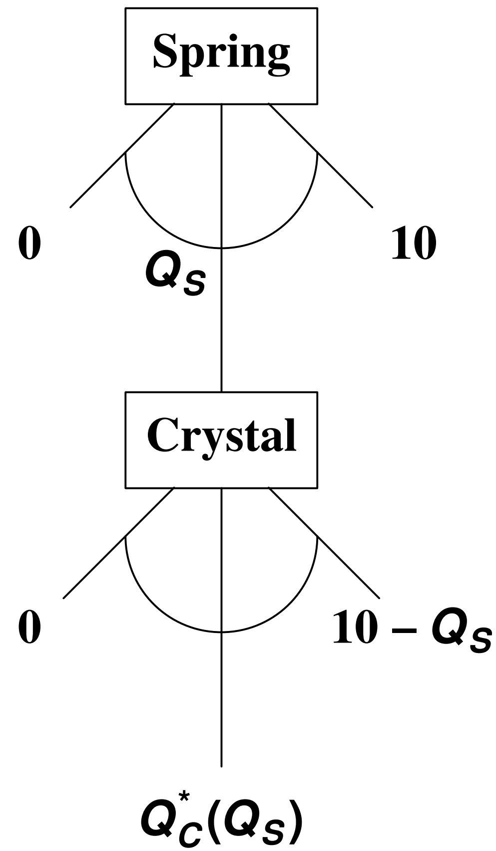
$$\pi_i(Q_S, Q_C) = (10 - Q_S - Q_C)Q_i - 3Q_i$$

- **zero output → zero profit**

Questions:

- **What should the Leader (Spring) do?**
  Depends on how the Leader thinks the Follower (Crystal) will react.

  Spring should: *Look forward and reason back.*
- **for every fixed level of output for Spring Q_S, what is the best (profit-maximising) level that Crystal can do? What is the Follower's *reaction function*?**



Cournot Rivalry Game Tree

(Quant. ⇨ Cournot,
Price ⇨ Bertrand.)

So: **Crystal (the Follower): chooses Q_C^* to max his profit:**

$$\pi_C = P(Q_S + Q_C)\,Q_C - TC_C(Q_C)$$
$$= (10 - (Q_S + Q_C))\,Q_C - 3\,Q_C$$

- **From Crystal's viewpoint, Spring's (the Leader's) output is predetermined — a constant Q_S, because Spring will already have moved first,**

  so Crystal sets $MR_C(Q_S, Q_C) = MC(Q_C) = 3$ to get Q_C^*:

  $$TR_C = (10 - Q_S - Q_C) \times Q_C = 10Q_C - Q_S Q_C - Q_C^2$$
  $$\therefore\ MR_C = 10 - Q_S - 2Q_C^* = MC_C = 3$$
  $$\rightarrow Q_C^* = f_C(Q_S) = \frac{7 - Q_S}{2}.$$

  the profit-maximising output Q_C^* of Follower (Crystal) is a function of the Leader's choice Q_S

- **This function is known as the *reaction function,* since it tells us how the Follower will react to the Leader's choice (of output in this case, but it could be price).**

So:

- **Solve for $Q_C^* = f_C(Q_S)$: $Q_C^* = \frac{7 - Q_S}{2}$,**
  Crystal's *reaction function*, contingent on Spring's action Q_S.
  Note: when $Q_S = 7$, then $Q_C^* = 0$.
- **Looking forward & reasoning back, Spring knows this, so Spring's profit:**

$$\pi_S = (10 - (Q_S + \frac{7 - Q_S}{2}))\,Q_S - 3Q_S$$
$$= 10Q_S - Q_S^2 - 3\tfrac{1}{2}\,Q_S + \tfrac{1}{2}\,Q_S^2 - 3Q_S$$
$$= 3\tfrac{1}{2}\,Q_S - \tfrac{1}{2}\,Q_S^2$$

- **Solving for maximum π_S (i.e. $\frac{d\pi}{dQ_S} = 0$) gives:**

$$\frac{d\pi}{dQ_S} = 3\tfrac{1}{2} - Q_S^* = 0$$

$\therefore$ π_S is max when $Q_S^* = 3.5$, $Q_C^* = 1.75$, $\therefore$ $Q = 5.25$,
with $\pi_S^* =$ \$6.125 and P =\$4.75
and $\pi_C^* =$ \$3.06

First-Mover Advantage:

- So Spring's *first-mover advantage* is the difference between Spring's profit π_S and Crystal's profit π_C:

$$\$6.125 - \$3.06 = \$3.06$$

- leadership — first mover
- leadership — innovator, monopolist, faced with threat of entry
- incumbent erects barriers to entry by new-comer
- long-term contracts reduce incumbent's flexibility and increase the credibility of defence

What Would A Monopolist's Outcomes Be?

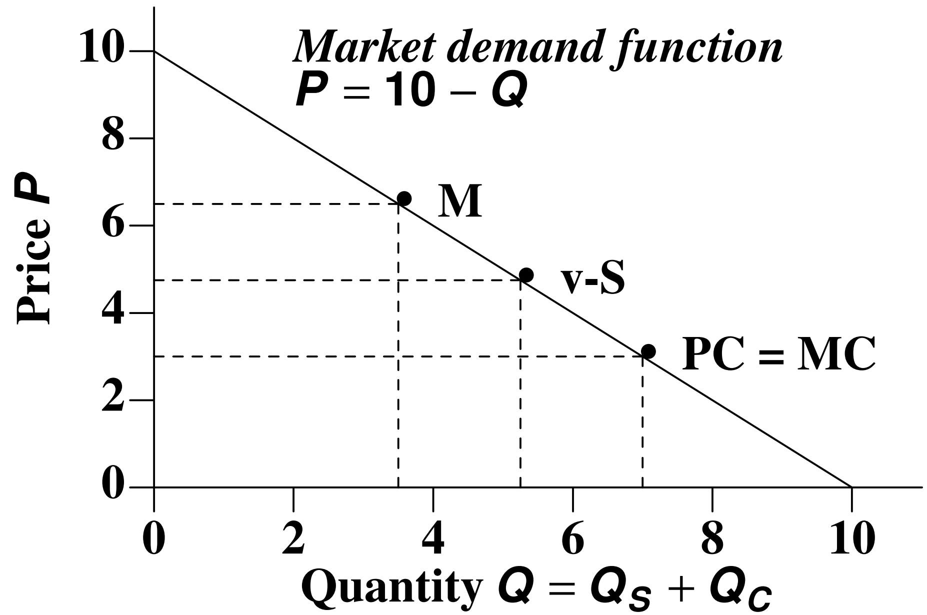
- **The monopolist quantity in this case is**

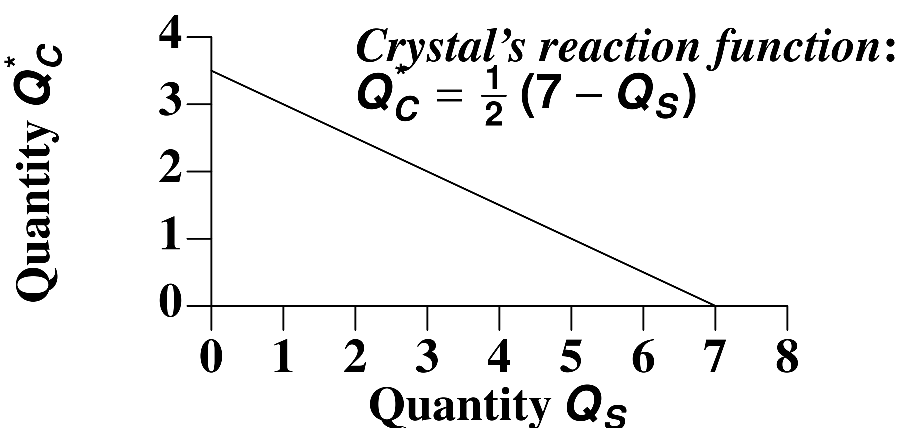
  Q_M = 3.5, with P_M = \$6.50

  and profit $\pi_M = 6.5 \times 3.5 - 3 \times 3.5 = \$12.25$

  This means that the Leader could become the Monopolist by paying the Follower not to enter the market, and offering him his (Follower's) profit of \$3.06 (π_C) not to, and still be ahead by:
  \$12.25 − 3.06 − 6.125 = \$3.06.

  I.e., $\pi_M - \pi_C - \pi_S = \$3.06$





i.e. as Q_S rises, Q_C^* falls, & vice versa: *strategic substitutes*.

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2. Simultaneous Cournot Quantity.

How much will each firm produce if they move simultaneously?

Each needs to:

1. **make a *conjecture or belief about how much the other firm will produce* — will it be high, with a lower industry price? Or vice versa?**
2. **then determine *its own quantity to produce* — balancing the gain from selling more units against the sacrifice of a lower price.**

There will be an industry-wide equilibrium when both firms resolve this balance.

From Spring's point of view, what should Q_S be?

- **"If Crystal produces Q_C, then I, Spring, should max my profit π_S:**
  $\pi_S = (10 - Q_C - Q_S)Q_S - 3Q_S$
- **which is maximised at**
  $Q_S^* = \frac{1}{2}(10 - 3 - Q_C) = \frac{1}{2}(7 - Q_C)$ **"**

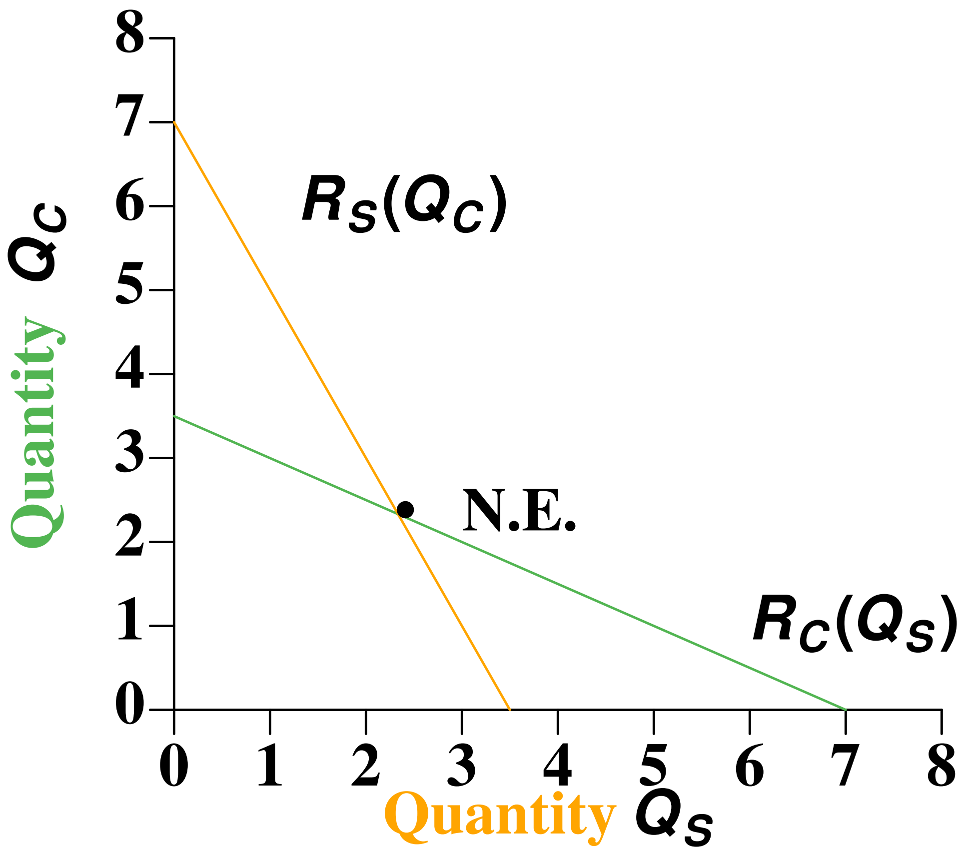
  So Q_S^* is Spring's *best response* to Crystal's quantity choice of Q_C, or $Q_S^* = R_S(Q_C)$.
- **If Q_C equals 7 units, then Spring should produce zero.**

Symmetrically, Crystal's *best response* $R_C(Q_S)$ to a conjectured production level of Q_S from Spring should be:

$$Q_C^* = \frac{1}{2}(7 - Q_S)$$

Reaction Functions:

Plotting both best responses, or *reaction functions*:



The Two Best-Reponse Curves: Cournot

Strategic substitutes: higher Q_1 ☞ lower Q_2

(Nash) Equilibrium.

There is a unique pair of quantities at which the two reaction functions cross: (Q_S^*, Q_C^*).

Hence at this point:

$$R_C(Q_S^*) = Q_C^* = 2\tfrac{1}{3} \text{ and}$$
$$R_S(Q_C^*) = Q_S^* = 2\tfrac{1}{3}$$

This is a so-called *Cournot N.E.*, where each player's conjecture is consistent with the other's actual production, and neither has any incentive to alter production. Their beliefs are fulfilled.

Price/unit = \$$5\tfrac{1}{3}$, profit of each = \$5.44

3. Simultaneous Price (Bertrand) Competition, Imperfect Substitutes.

Two pizzerias — Donna's Deep Dish and Perce's Pizza Pies — simultaneously compete in a small town.

When Donna's price is P_d and Perce's price is P_p, then their sales Q_d and Q_p (hundreds per week) are:

$$Q_d = 24 - P_d + \tfrac{1}{2} P_p$$
$$Q_p = 24 - P_p + \tfrac{1}{2} P_d$$

The two brands of pizza are (*imperfect*) *substitutes*:
if the price of one rises, sales of the other go up:
half the discouraged buyers switch to the other pizza and half move to some other kind of food (Big Macs?)

It costs $6 to make each pizza.

Perce sets his price P_p to maximise his profits:

$$\pi_p = (P_p - 6)\,Q_p = (P_p - 6)\,(24 - P_p + \tfrac{1}{2}\,P_d)$$

Perce's best price P_p for each level of his conjecture about Donna's price P_d will give his best response.
Expanding Perce's profit function:

$$\pi_p = -\,144 - 3P_d + (30 + \tfrac{1}{2}\,P_d)P_p - P_p^2$$

Diff. w. r. to P_p (holding Donna's price P_d constant):

$$\frac{d\pi_p}{dP_p} = 30 + \tfrac{1}{2}\,P_d - 2\,P_p.$$

The first-order condition for P_p to max π_p: $\frac{d\pi_p}{dP_p} = 0$.
∴ Perce's best-response curve $R_p(P_d)$ is:

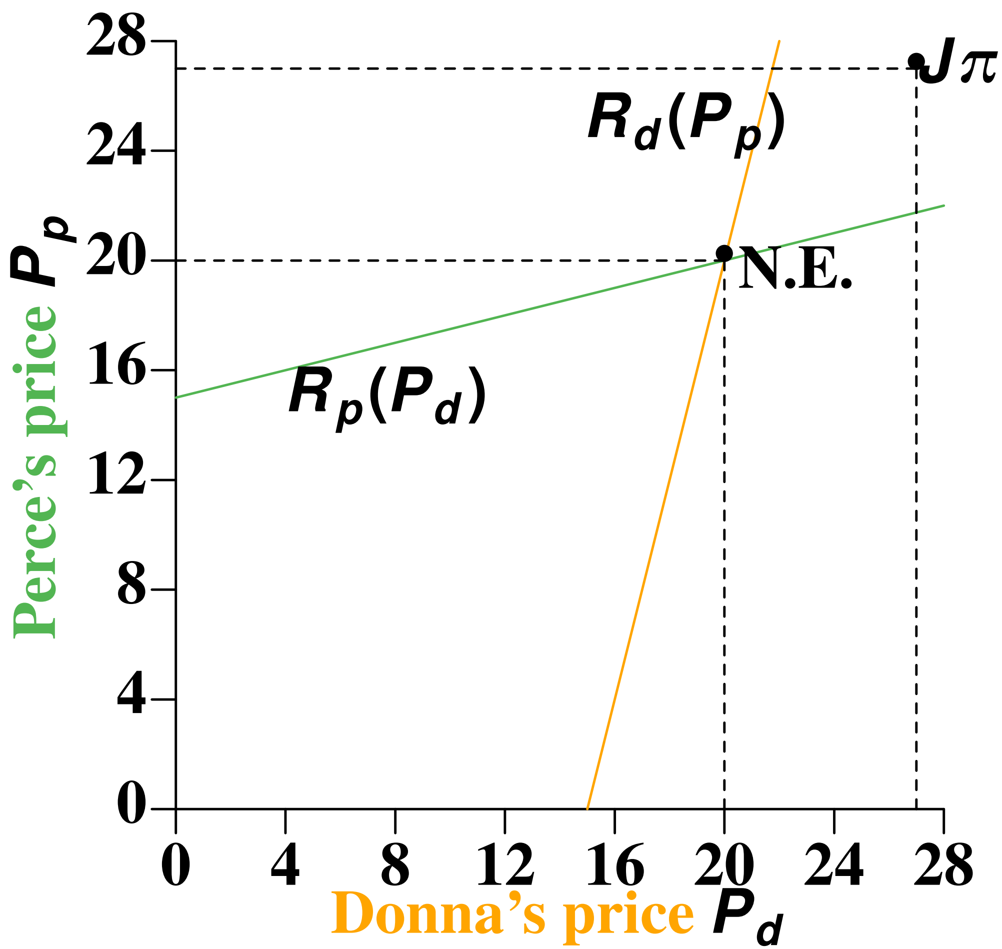
$$P_p = 15 + ¼\,P_d$$

∴ if Donna's price is \$32, then Perce's best price is \$23.

Symmetrically, Donna's best-response curve $R_d(P_p)$ is:

$$P_d = 15 + ¼\,P_p$$

Plotting the Response Curves:



The Two Best-Response Curves (Reaction Functions): Bertrand

(*Strategic complements:*, as P_p rises, so does P_d.)

The two reaction curves intersect at the N.E. of the shops' pricing game, at $P_d = P_p =$ \$20.

Then each shop sells 1400 pizzas a week, at a profit per shop of \$19,600 per week.

The best-response reaction curves slope up: when one shop raises its price by \$1, then its rival should increase its price by 25¢: when the first shop raised its price, then some of its customers switched to its rival, which could then best profit from the new customers by raising its price somewhat.

∴ a shop that increases its price is helping increase the profits of its rival, but this side-effect is uncaptured (and so ignored) by each shop independently.

But, as in the PD, they could collude and increase their profits: if they each charged, say, \$24, then each would sell 1200 pizzas, for a profit of \$21,600.

But if P_d = \$24, then Perce's best response would be \$21, to which Donna would respond with \$20.25, etc., etc, until they converge to the N.E. prices of \$20.

What is best for them jointly?

Let P_d = P_p = P (they both charge P, because the market demands are symmetric) and π is maximised when P = \$27; at which price each sells 1050 pizzas, at a profit of \$22,050 per shop.

Shown as $J\pi$ on the figure above.

Note: if the market demands were *not* symmetric, then it would be wrong to charge the same price P for both pizzas. Need to choose the two prices to max $\pi = \pi_d + \pi_p$.

Which form of competition: Cournot (quantity) or Bertrand (price)?

What sort of industries compete using quantity and what sort by price? Can they choose?

***Cournot*: Changing prices can be costly, but so can changing the flow of production to increase or cut back the quantity of good flowing to the market.**

***Cournot* applies to markets in which firms must make production decisions in advance and face high costs for holding inventories: price will adjust more quickly than quantity, and each firm will set a price that sells all that it produces.**

Such a firm will not expect to steal customers by lowering its price, since its rivals will immediately match any price change, so that their sales equal their planned production volumes. Hence there is less competition than Betrand.

When firms choose capacity first, and then compete as price setters given the capacities chosen earlier, then the result is identical to Cournot quantity competition.

Examples?

***Bertrand*: When capacity is sufficiently flexible to allow them to adjust production to meet all the demand for their product at any price they announce, then *Bertrand* price competition results. With perfect substitutes, firms will attempt to steal customers by price cutting down to marginal cost.**

Examples?

4. Strategic Complements and Substitutes

Compare the reaction functions in the Cournot (quantity) game between the water sellers and those in the Bertrand (price) game between the pizzerias.

In the Cournot model the reaction functions were downwards sloping,
and in the Bertrand model they were upwards sloping.

Strategic complements:

in general, when reaction functions (best-response curves) are *upwards sloping*, then we say that the firms' *actions* (here, pizza prices) are *strategic complements*.

Strategic substitutes:

in general, when reaction functions are *downwards sloping*, then we say that the firms' *actions* (here, water quantities) are *strategic substitutes*.

Meaning?

When actions are *strategic complements*, then an increase by one firm will elicit an increase by the other, & v.v.

> **In the Bertrand (price) model, prices are strategic complements: a competitor's price cut will best be responded to by a price cut. $\rightarrow$ price wars.**

When actions are *strategic substitutes*, then an increase by one firm will elicit a reduction by the other, & v.v.

> **In the Cournot (quantity) model, quantities are strategic substitutes: a competitor's quantity increase will best be responded to by a quantity cut.**

***A rule of thumb*: quantities and capacity decisions almost always strategic substitutes, whereas prices almost always strategic complements.**

So what?

So What?

How will a firm expect its rivals to react to its tactical manœuvres?

- **When actions are strategic complements, then increased aggression will elicit increased aggression in the rival: (Bertrand)**

  e.g. pizza prices: a competitor's price cut (an aggressive move) will best be responded to by a price cut (also an aggressive move), since the reaction functions are upwards sloping.

- **When actions are strategic substitutes, then increased aggression will result in lessened aggression in the rival: (Cournot)**

  e.g. water quantities: a competitor's quantity increase (aggressive) will best be responded to by a quantity cut (a soft response), since the reaction functions are downwards sloping.

Compare the Cournot and Bertrand duopoly profits below.

5. Benchmarking Oligopoly Behaviour

Two companies produce homogeneous output.

Linear industry demand curve of $P = 10 - Q$, where Q is the sum of the two companies' outputs, $Q = y_1 + y_2$.

Both companies have identical costs, $AC = MC = \$1$/unit.

Five possibilities for the equilibrium levels of prices, outputs, and profits:

1. **Competitive Price Taking**
2. **Monopolistic Cartel**
3. **Cournot Oligopolists**
4. **von Stackelberg Quantity Leadership**
5. **Bertrand Simultaneous Price Setting.**

1 Competitive Price Taking

Each setting price equal to marginal cost.

So: Price P_{PC} = \$1/unit,
the total quantity Q = 9 units between them,
and each produces output $y_1 = y_2 = 4.5$ units.

Since P_{PC} = AC, their profits are zero: $\pi_1 = \pi_2 = 0$.

2 Monopolistic Cartel

Collude and act as a monopolistic *cartel.*

Each produces half of the monopolist's output and receives half the monopolist's profit.

Output Q_M such that $MR(Q_M) = MC$ = \$1/unit.

The MR curve is given by $MR = 10 - 2Q$, which results in:

Q_M = 4.5 units,
P_M = \$5.5/unit, and
π_M = (5.5 − 1)×4.5 = \$20.25.

Each produces output $y_1 = y_2 =$ 2.25 units, and earns $\pi_1 = \pi_2$ = \$10.125 profit.

3 Cournot Oligopolists

Each chooses its output to maximise its profit, assuming that the other is doing likewise: not colluding, but competing. They choose simultaneously.

Cournot N.E. occurs where their *reaction curves* intersect and the expectations of each are fulfilled.

Firm 1 determines Firm 2's reaction function: "If I were Firm 2, I'd choose my output y_2^* to maximise my Firm 2 profit conditional on the expectation that Firm 1 produced output of y_1^e."

$$\max_{y_2} \pi_2 = (10 - y_2 - y_1^e) \times y_2 - y_2$$

Thus $y_2 = \frac{1}{2}(9 - y_1^e)$, which is Firm 2's reaction function, given its conjecture y_1^e of Firm 1's behaviour.

Since the two firms are identical, Cournot equilibrium occurs where the two reaction curves intersect, at $y_1^* = y_1^e = y_2^* = y_2^e = 3$ units.

So $Q_{Co} = 6$ units, price P_{Co} is then \$4/unit, and the profit of each firm is \$9.

4 von Stackelberg Quantity Leadership

What if Firm 1 gets to choose its output level y_1 first?

It realises that Firm 2 will know Firm 1's output level when Firm 2 chooses its level: see Firm 2's reaction function from 3. above, but with the actual, not the expected, level of Firm 1's output, y_1.

So Firm 1 chooses y_1^* to maximise its profit:

$$\max_{y_1} \pi_1 = (10 - y_2 - y_1) \times y_1 - y_1,$$

where Firm 2's output y_2 is given by Firm 2's reaction function: $y_2 = \frac{1}{2}(9 - y_1)$.

Substituting this into Firm 1's maximisation problem:

y_1^* = 4.5 units, and so y_2^* = 2.25 units, so that Q_{St} = 6.75 units and P_{St} = \$3.25/unit.

Profits are π_1 = \$10.125 (the same as in the cartel case 2. above) and π_2 = \$5.063 (half the cartel profit).

5 Bertrand Simultaneous Price Setting.

Remember: Equilibrium means there is no incentive for either firm to undercut the other.

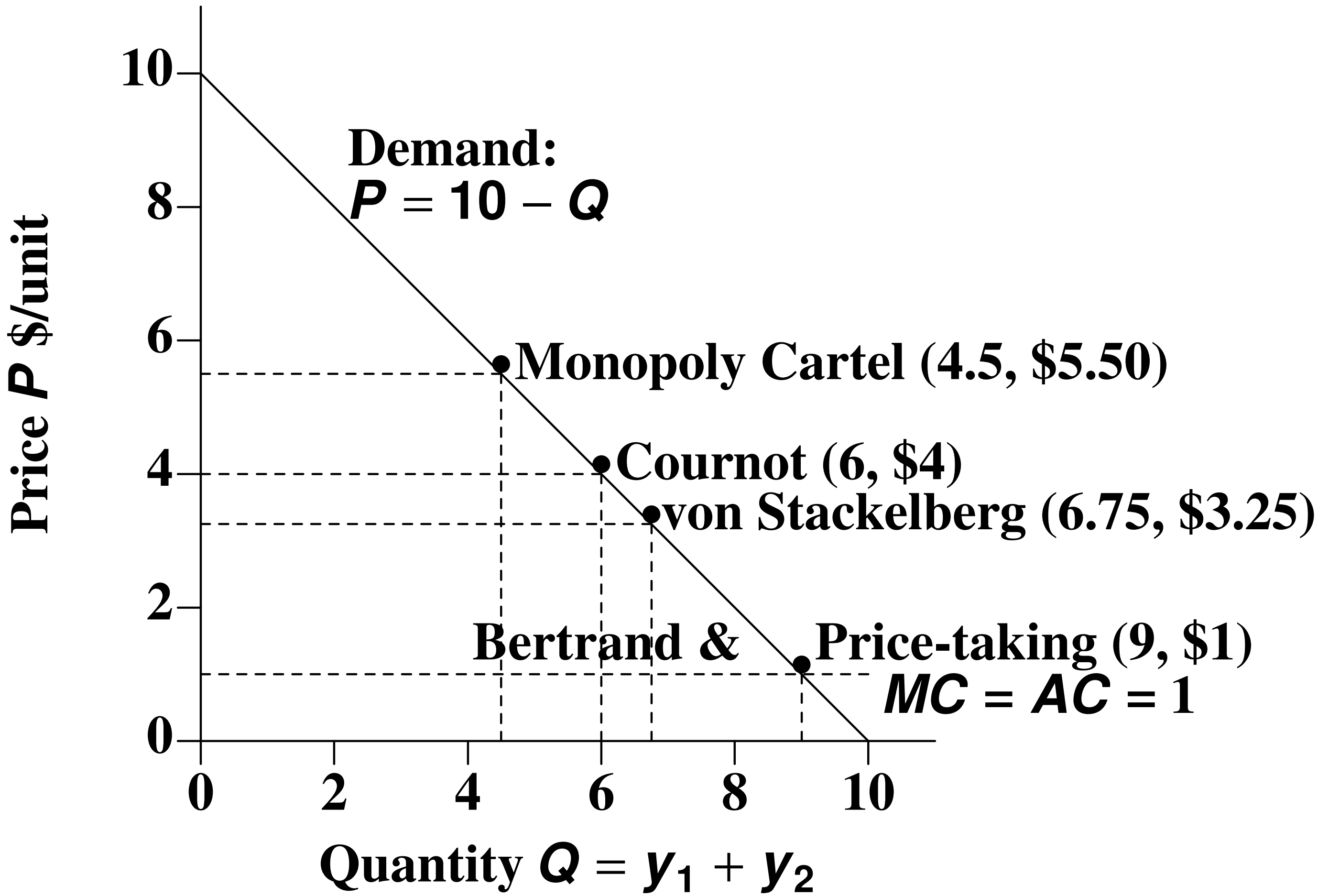
The only equilibrium when they compete using price is where each is selling at $P_1 = P_2 = MC_1 = MC_2 = \$1$/unit.

Identical to the price-taking case above.

***Note:* Were MC_1 greater than MC_2, then Firm 2 would capture the whole market at a price just below MC_1, and would make a positive profit; and $y_1 = 0$.**

Market	Output y_1	Profit π_1	Output y_2	Profit π_2	Price P	Quantity $Q = y_1 + y_2$
1 Price-taking	4.5	0	4.5	0	1	9
2 Cartel	2.25	10.125	2.25	10.125	5.5	4.5
3 Cournot	3	9	3	9	4	6
4 von Stackelberg	4.5	10.125	2.25	5.063	3.25	6.75
5 Bertrand	4.5	0	4.5	0	1	9

Summary of Outcomes.



10
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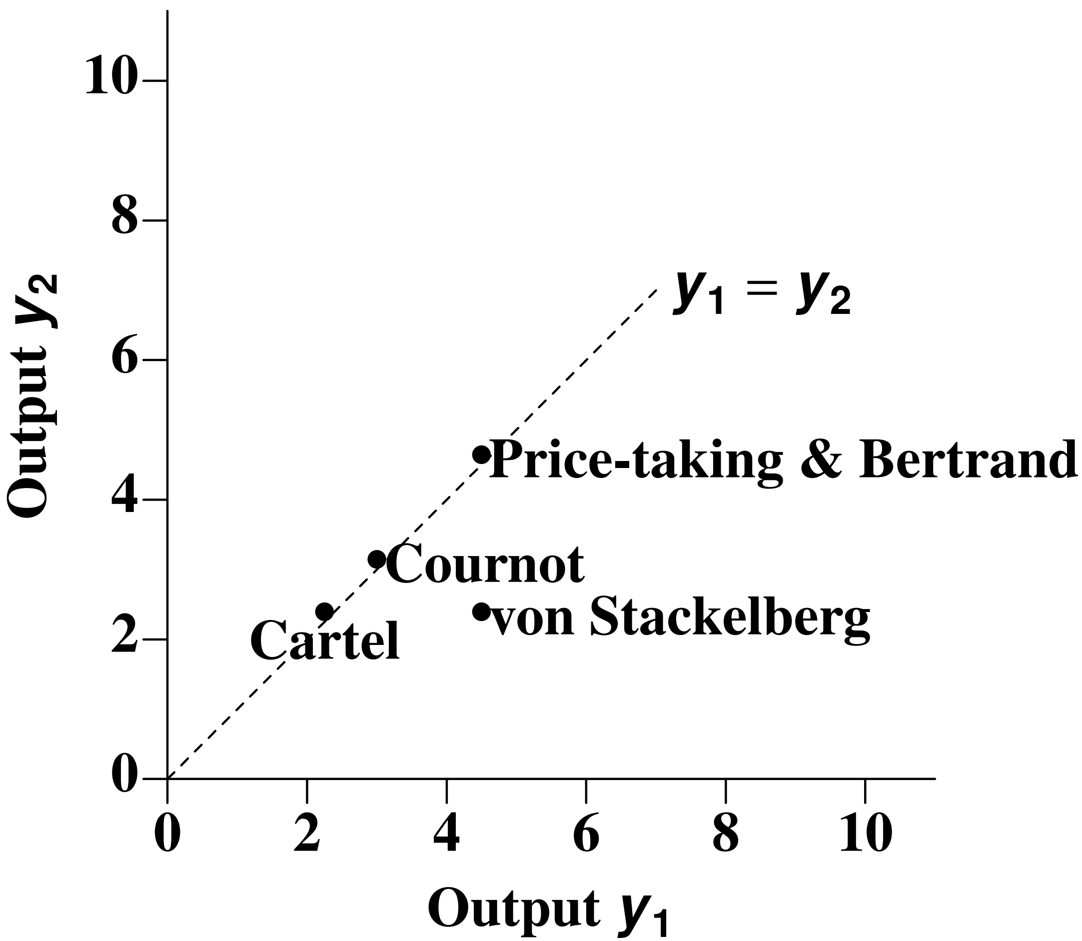
Price P $/unit
Quantity Q = y1 + y2
Demand:
P = 10 − Q
Monopoly Cartel (4.5, $5.50)
Cournot (6, $4)
von Stackelberg (6.75, $3.25)
Bertrand &
Price-taking (9, $1)
MC = AC = 1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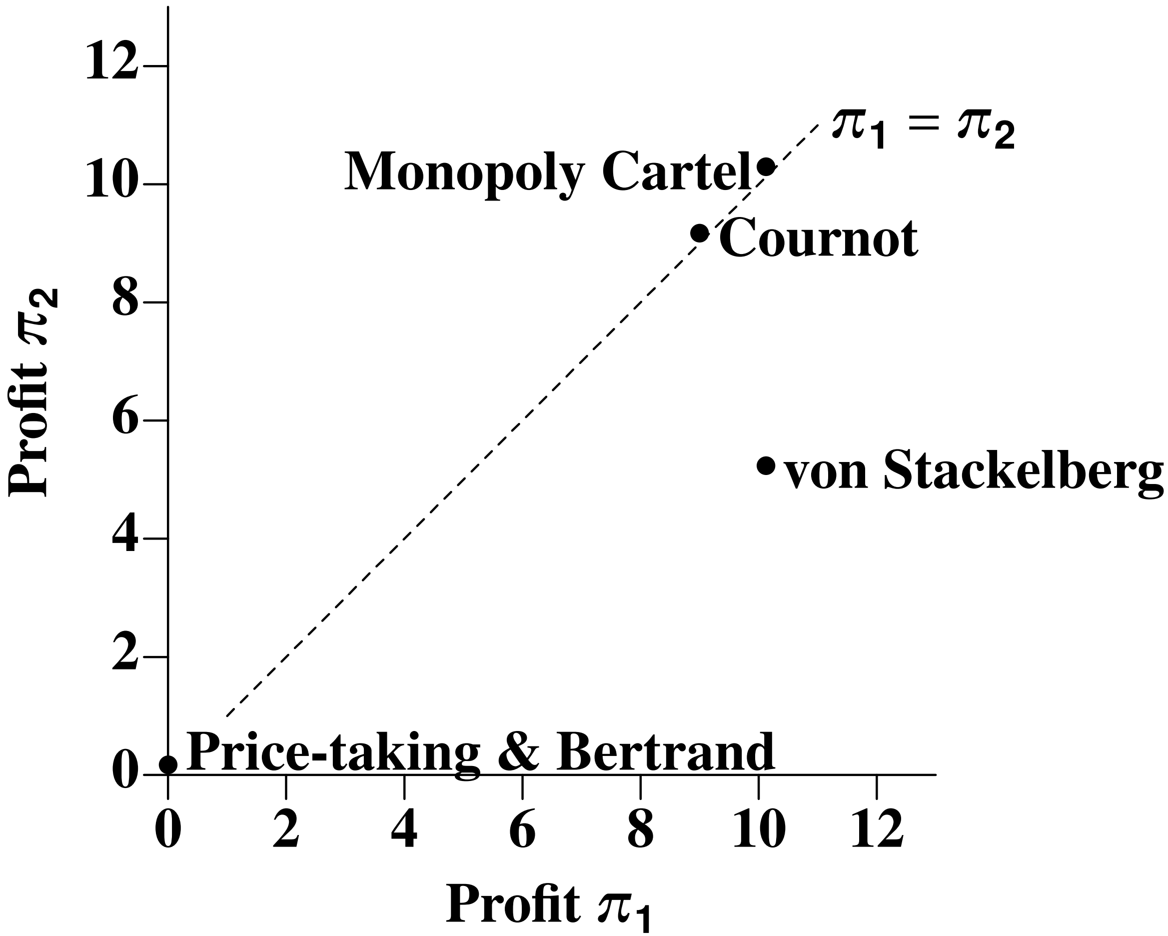
Output y_2
Output y_1
$y_1 = y_2$
Price-taking & Bertrand
Cournot
von Stackelberg
Cartel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$\pi_1 = \pi_2$
Monopoly Cartel
Cournot
von Stackelberg
Price-taking & Bertrand
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Profit π_1

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